Vehicle Fundamentals
Forces acting on a vehicle

Newton’s second law for vehicle traction

\[
\frac{dV}{dt} = \frac{\Sigma F_t - \Sigma F_{tr}}{\delta M_v}
\]

- \( F_t \): traction force
- \( F_{tr} \): resistance force
- \( M_v \): total mass
- \( \delta \): mass factor
- \( V \): vehicle speed
Vehicle Resistance

Vehicle resistance opposing its movement includes rolling resistance of the tires rolling resistance torque $T_{rf}$ and $T_{rr}$, aerodynamic drag $F_w$, and grading resistance ($M_v g \sin \alpha$)

Rolling Resistance

Tire deflection and rolling resistance on a (a) hard and (b) soft road surface
The rolling resistant moment;

\[ T_r = Pa \]

To keep the wheel rolling, a force \( F \), acting on the center of the wheels, is required to balance this rolling resistant moment. This force is expressed as

\[ F = \frac{T_r}{r_d} = \frac{Pa}{r_d} = Pf_r \]

where \( r_d \) is the effective radius of the tire and \( f_r = a/r_d \) is called the rolling resistance coefficient.

\[ F_r = Pf_r \]

The equivalent force is called rolling resistance with a magnitude of

\[ F_r = Pf_r \cos \alpha \]

where \( P \) is the normal load, acting on the center of the rolling wheel. When a vehicle is operated on a slope road, the normal load, \( P \), should be replaced by the component, which is perpendicular to the road surface. That is,

\[ F_r = Pf_r \cos \alpha \]

where \( \alpha \) is the road angle.
The values given in above Table do not take into account their variations with speed. Based on experimental results, many empirical formulae have been proposed for calculating the rolling resistance on a hard surface.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Rolling resistance coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car tires on concrete or asphalt</td>
<td>0.013</td>
</tr>
<tr>
<td>Car tires on rolled gravel</td>
<td>0.02</td>
</tr>
<tr>
<td>Tar macadam</td>
<td>0.025</td>
</tr>
<tr>
<td>Unpaved road</td>
<td>0.05</td>
</tr>
<tr>
<td>Field</td>
<td>0.1–0.35</td>
</tr>
<tr>
<td>Truck tires on concrete or asphalt</td>
<td>0.006–0.01</td>
</tr>
<tr>
<td>Wheels on rail</td>
<td>0.001–0.002</td>
</tr>
</tbody>
</table>
For example; the rolling resistance coefficient of passenger cars on concrete road may be calculated from the following equation:

\[ f_r = f_0 + f_s \left( \frac{V}{100} \right)^{2.5} \]

where \( V \) is vehicle speed in km/h, and \( f_0 \) and \( f_s \) depend on inflation pressure of the tire.

Range of inflation pressure:

\[ f_r = 0.01 \left( 1 + \frac{V}{100} \right) \]

This equation predicts the values of \( f_r \) with acceptable accuracy for speeds up to 128 km/h.
Aerodynamic Drag

A vehicle traveling at a particular speed in air encounters a force resisting its motion. This force is referred to as aerodynamic drag. It mainly results from two components: shape drag and skin friction.

*Shape drag:* The forward motion of the vehicle pushes the air in front of it.
**Skin friction**: Air close to the skin of the vehicle moves almost at the speed of the vehicle while air far from the vehicle remains still. In between, air molecules move at a wide range of speeds. The difference in speed between two air molecules produces a friction that results in the second component of aerodynamic drag. Aerodynamic drag is a function of vehicle speed $V$, vehicle frontal area $A_f$, shape of the vehicle, and air density $\rho$. Aerodynamic drag is expressed as

$$F_w = \frac{1}{2} \rho A_f C_D (V + V_\omega)^2$$

where $C_D$ is the aerodynamic drag coefficient that characterizes the shape of the vehicle and $V_\omega$ is the component of wind speed on the vehicle’s moving direction, which has a positive sign when this component is opposite to the vehicle speed and a negative sign when it is in the same direction as vehicle speed.
The aerodynamic drag coefficients for a few types of vehicle body shapes are shown in Figure.

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Coefficient of Aerodynamic Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open convertible</td>
<td>0.5–0.7</td>
</tr>
<tr>
<td>Van body</td>
<td>0.5–0.7</td>
</tr>
<tr>
<td>Ponton body</td>
<td>0.4–0.55</td>
</tr>
<tr>
<td>Wedge-shaped body; headlamps and bumpers are integrated into the body, covered underbody, optimized cooling air flow</td>
<td>0.3–0.4</td>
</tr>
<tr>
<td>Headlamp and all wheels in body, covered underbody</td>
<td>0.2–0.25</td>
</tr>
<tr>
<td>K-shaped (small breakaway section)</td>
<td>0.23</td>
</tr>
<tr>
<td>Optimum streamlined design</td>
<td>0.15–0.20</td>
</tr>
<tr>
<td>Trucks, road trains</td>
<td>0.8–1.5</td>
</tr>
<tr>
<td>Buses</td>
<td>0.6–0.7</td>
</tr>
<tr>
<td>Streamlined buses</td>
<td>0.3–0.4</td>
</tr>
<tr>
<td>Motorcycles</td>
<td>0.6–0.7</td>
</tr>
</tbody>
</table>
Grading Resistance

When a vehicle goes up or down a slope, its weight produces a component, which is always directed to the downward direction, as shown in figure.

In vehicle performance analysis, only uphill operation is considered. This grading force is usually called grading resistance. The grading resistance, from above Figure, can be expressed as

\[ F_g = M_y g \sin \alpha \]
The road angle, $\alpha$, is usually replaced by grade value when the road angle is small. The grade is defined as

$$i = \frac{H}{L} = \tan \alpha \approx \sin \alpha$$

In some literature, the tire rolling resistance and grading resistance together are called road resistance, which is expressed as

$$F_{rd} = F_f + F_g = M_v g (f_r \cos \alpha + \sin \alpha)$$

When the road angle is small, the road resistance can be simplified as

$$F_{rd} = F_f + F_g = M_v g (f_r + i)$$
Dynamic Equation

In the longitudinal direction, the major external forces acting on a two-axle vehicle, include the rolling resistance of front and rear tires \( F_{rf} \) and \( F_{rr} \), which are represented by rolling resistance moment \( T_{rf} \) and \( T_{rr} \), aerodynamic drag \( F_{\omega} \), grading resistance \( F_{g} (M_{v} g \sin \alpha) \), and tractive effort of the front and rear tires, \( F_{tf} \) and \( F_{tr} \). \( F_{tf} \) is zero for a rear-wheel-driven vehicle, whereas \( F_{tr} \) is zero for a front-wheel-driven vehicle. The dynamic equation of vehicle motion along the longitudinal direction is expressed by

\[
M_{v} \frac{dV}{dt} = (F_{tf} + F_{tr}) - (F_{rf} + F_{rr} + F_{\omega} + F_{g})
\]

where \( dV/dt \) is the linear acceleration of the vehicle along the longitudinal direction and \( M_{v} \) is the vehicle mass.

By summing the moments of all the forces about point \( R \) (center of the tire–ground area), the normal load on the front axle \( W_{f} \) can be determined as

\[
W_{f} = \frac{M_{v} g L_{b} \cos \alpha - (T_{tf} + T_{rr} + F_{\omega} h_{w} + M_{v} g h_{g} \sin \alpha + M_{h} g dV/dt)}{L}
\]
Similarly, the normal load acting on the rear axle can be expressed as

\[
W_r = \frac{M_v g L_a \cos \alpha - (T_{rf} + T_{rr} + R_w h_w + M_v g h_g \sin \alpha + M_v h_g \frac{dV}{dt})}{L}
\]

For passenger cars, the height of the center of application of aerodynamic resistance, \( h_\omega \), is assumed to be near the height of the center of gravity of the vehicle, \( h_g \).

\[
W_f = \frac{L_b}{L} M_v g \cos \alpha - \frac{h_g}{L} \left( F_w + F_g + M_v g f_r h_g r_d \cos \alpha + M_v \frac{dV}{dt} \right)
\]

\[
W_r = \frac{L_a}{L} M_v g \cos \alpha + \frac{h_g}{L} \left( F_w + F_g + M_v g f_r h_g r_d \cos \alpha + M_v \frac{dV}{dt} \right)
\]

where \( r_d \) is the effective radius of the wheel.

\[
W_f = \frac{L_b}{L} M_v g \cos \alpha - \frac{h_g}{L} \left( F_i - F_r \left( 1 - \frac{r_d}{h_g} \right) \right)
\]

\[
W_r = \frac{L_a}{L} M_v g \cos \alpha + \frac{h_g}{L} \left( F_i - F_r \left( 1 - \frac{r_d}{h_g} \right) \right)
\]

where \( F_t = F_{rf} + F_{tr} \) is the total tractive effort of the vehicle and \( F_r \) is the total rolling resistance of the vehicle.
\[ F_{l_{\text{max}}} = \mu W_f = \mu \left[ \frac{L_b}{L} M_v g \cos \alpha - \frac{h_g}{L} \left( F_{l_{\text{max}}} - F_r \left( 1 - \frac{r_d}{h_g} \right) \right) \right] \]

\[ F_{l_{\text{max}}} = \frac{\mu M_v g \cos \alpha [L_b + f_r (h_g - r_d)] / L}{1 + \mu h_g / L} \]

where \( f_r \) is the coefficient of the rolling resistance, coefficient of road adhesion \( \mu \).

For a rear-wheel-driven vehicle,

\[ F_{l_{\text{max}}} = \mu W_r = \mu \left[ \frac{L_a}{L} M_v g \cos \alpha - \frac{h_g}{L} \left( F_{l_{\text{max}}} - F_r \left( 1 - \frac{r_d}{h_g} \right) \right) \right] \]

\[ F_{l_{\text{max}}} = \frac{\mu M_v g \cos \alpha [L_a + f_r (h_g - r_d)] / L}{1 + \mu h_g / L} \]
The slip, $s$, of a tire is usually defined as

$$s = \left(1 - \frac{V}{r\omega}\right) \times 100\% = \left(1 - \frac{r_e}{r}\right) \times 100\%$$

where $V$ is the translatory speed of the tire center, $\omega$ is the angular speed of the tire, $r$ is the rolling radius of the free rolling tire, and $r_e$ is the effective rolling radius of the tire, defined as the ratio of the translatory speed of the tire center to the angular speed of the tire. In traction, the speed $V$ is less than $r\omega$, therefore, the slip of the tire has a positive value between 0 and 1.0. During braking, however, the tire slip would be defined as

$$s = \left(1 - \frac{r\omega}{V}\right) \times 100\% = \left(1 - \frac{r}{r_e}\right) \times 100\%$$

which has a positive value between 0 and 1.0, similar to traction. The maximum traction effort of a tire corresponding to a certain tire slip is usually expressed as

$$F_x = P\mu$$

where $P$ is the vertical load of the tire and $\mu$ is the tractive effort coefficient, which is a function of tire slip.
The tractive effort coefficient and tire slip always have a relationship as shown in Figure.
Behavior of a tire under the action of driving torque
For normal driving, the slip of the tire must be limited in a range less than 15–20%. Under
the Table shows the average values of tractive effort coefficients on various roads.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Peak Values, $\mu_p$</th>
<th>Sliding Values, $\mu_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asphalt and concrete (dry)</td>
<td>0.8–0.9</td>
<td>0.75</td>
</tr>
<tr>
<td>Concrete (wet)</td>
<td>0.8</td>
<td>0.7</td>
</tr>
<tr>
<td>Asphalt (wet)</td>
<td>0.5–0.7</td>
<td>0.45–0.6</td>
</tr>
<tr>
<td>Grave</td>
<td>0.6</td>
<td>0.55</td>
</tr>
<tr>
<td>Earth road (dry)</td>
<td>0.68</td>
<td>0.65</td>
</tr>
<tr>
<td>Earth road (wet)</td>
<td>0.55</td>
<td>0.4–0.5</td>
</tr>
<tr>
<td>Snow (hard packed)</td>
<td>0.2</td>
<td>0.15</td>
</tr>
<tr>
<td>Ice</td>
<td>0.1</td>
<td>0.07</td>
</tr>
</tbody>
</table>
Power Train Tractive Effort and Vehicle Speed

Conceptual illustration of an automobile power train;
The torque on the driven wheels, transmitted from the power plant, is expressed as

\[ T_w = i_g i_0 \eta_t T_p \]

where \( i_g \) is the gear ratio of the transmission defined as \( i_g = \frac{N_{in}}{N_{out}} \) (\( N_{in} \): input rotating speed, \( N_{out} \): output rotating speed), \( i_0 \) is the gear ratio of the final drive, \( \eta_t \) is the efficiency of the driveline from the power plant to the driven wheels, and \( T_p \) is the torque output from the power plant. The tractive effort on the driven wheels, as shown in Figure, can be expressed as

\[ F_t = \frac{T_w}{r_d} \]

\[ F_t = \frac{T_p i_g i_0 \eta_t}{r_d} \]
The friction in the gear teeth and the friction in the bearings create losses in mechanical gear transmission. The following are representative values of the mechanical efficiency of various components:

- Clutch: 99%
- Each pair of gears: 95–97%
- Bearing and joint: 98–99%

The total mechanical efficiency of the transmission between the engine output shaft and drive wheels or sprocket is the product of the efficiencies of all the components in the driveline. As a first approximation, the following average values of the overall mechanical efficiency of a manual gear-shift transmission may be used:

- Direct gear: 90%
- Other gear: 85%
- Transmission with a very high reduction ratio: 75–80%

The rotating speed (rpm) of the driven wheel can be expressed as

\[ N_w = \frac{N_p}{i_g i_0} \]

where \( N_p \) is the output rotating speed (rpm). The translational speed of the wheel center (vehicle speed) can be expressed as

\[ V = \frac{\pi N_w r_d}{30} \text{ (m/s)} \]

\[ V = \frac{\pi N_p r_d}{30 i_g i_0} \text{ (m/s)} \]
Vehicle Power Plant and Transmission Characteristics

Power Plant Characteristics

For vehicular applications, the ideal performance characteristic of a power plant is the constant power output over the full speed range. Consequently, the torque varies with speed hyperbolically as shown in ideal performance characteristics for a vehicle traction power plant Figure.
Since the internal combustion engine and electric motor are the most commonly used power plants for automotive vehicles to date, it is appropriate to review the basic features of the characteristics that are essential to predicating vehicle performance and driveline design. Representative characteristics of a gasoline engine in full throttle and an electric motor at full load are shown under figures.

Typical performance characteristics of gasoline engines

Typical performance characteristics of electric motors for traction
A multigear transmission is usually employed to modify it, as shown in Figure

![Graph showing tractive effort on wheel vs. vehicle speed](image1)

A single-gear or double-gear transmission is usually employed, as shown in Figure

![Graph showing tractive effort on wheel vs. speed](image2)
Transmission Characteristics

The transmission requirements of a vehicle depend on the characteristics of the power plant and the performance requirements of the vehicle. For automobile applications, there are usually two basic types of transmission: manual gear transmission and hydrodynamic transmission.

Manual Gear Transmission

Manual gear transmission consists of a clutch, gearbox, final drive, and drive shaft.

![Tractive effort characteristics of a gasoline engine-powered vehicle](image1)

![Demonstration of vehicle speed range and engine speed range for each gear](image2)
For a four-speed gearbox, the following relationship can be established

\[
\frac{i_{g1}}{i_{g2}} = \frac{i_{g2}}{i_{g3}} = \frac{i_{g3}}{i_{g4}} = K_g
\]

\[
K_g = \sqrt[3]{\frac{i_{g1}}{i_{g4}}}
\]

where \(i_{g1}, i_{g2}, i_{g3}, \) and \(i_{g4}\) are the gear ratios for the first, second, third, and fourth gear, respectively.

The number of the gear \(ng\) is known, the factor \(K_g\) can be determined as

\[
K_g = \left(\frac{i_{g1}}{i_{gn}}\right)^{n_g - 1}
\]

and each gear ratio can be obtained by

\[
\begin{align*}
i_{gn-1} &= K_g i_{gn} \\
i_{gn-2} &= K_g^2 i_{gn} \\
& \vdots \\
i_{g2} &= K_g^{n_g-1} i_{gn}
\end{align*}
\]

For passenger cars, to suit changing traffic conditions, the step between the ratios of the upper two gears is often a little closer than that based on upper equations. That is,

\[
\frac{i_{g1}}{i_{g2}} > \frac{i_{g2}}{i_{g3}} > \frac{i_{g3}}{i_{g4}}
\]
Hydrodynamic Transmission

Hydrodynamic transmissions use fluid to transmit power in the form of torque and speed and are widely used in passenger cars. They consist of a torque converter and an automatic gearbox. The torque converter consists of at least three rotary elements known as the impeller (pump), the turbine, and the reactor as shown in figure.
The major advantages of hydrodynamic transmission:

- When properly matched, the engine will not stall.
- It provides flexible coupling between the engine and the driven wheels.
- Together with a suitably selected multispeed gearbox, it provides torque–speed characteristics that approach the ideal.

The performance characteristics of a torque converter are described in terms of the following four parameters:

1. **Speed ratio**
   
   \[ C_{sr} = \frac{\text{output\_speed}}{\text{input\_speed}} \]

   which is the reciprocal of the gear ratio mentioned before.

2. **Torque ratio**

   \[ C_{tr} = \frac{\text{output\_torque}}{\text{input\_torque}} \]

3. **Efficiency**

   \[ \eta_e = \frac{\text{output\_speed} \times \text{output\_torque}}{\text{input\_speed} \times \text{input\_torque}} = C_{sr} C_{tr} \]

4. **Capacity factor (size factor)**

   \[ K_c = \frac{\text{speed}}{\text{torque}} \]
To characterize the engine operating condition for the purpose of determining the combined performance of the engine and the converter, an engine capacity factor, $K_e$, is introduced and defined as

$$K_e = \frac{n_e}{\sqrt{T_e}}$$

where $n_e$ and $T_e$ are engine speed and torque, respectively. The variation of the capacity factor with speed for a typical engine.
The engine shaft is usually connected to the input shaft of the torque converter, as mentioned above. That is,

\[ K_e = K_c \]

The matching procedure begins with specifying the engine speed and engine torque. Knowing the engine operating point, one can determine the engine capacity factor, \( K_e \).
For a particular value of the input capacity factor of the torque converter, $K_{tc}$, the converter speed ratio, $C_{sr}$, and torque ratio, $C_{tr}$, can be determined from the torque converter performance characteristics. The output torque and output speed of the converter are then given by

$$T_{tc} = T_e C_{tr}$$

$$n_{tc} = n_e C_{sr}$$

where $T_{tc}$ and $n_{tc}$ are the output torque and output speed of the converter, respectively.

$$F_t = \frac{T_e C_{tr} i_g i_0 \eta_t}{r}$$

$$V = \frac{\pi n_e C_{sr} r}{30 i_g i_0} \text{ (m/s)} = 0.377 \frac{n_e C_{sr} r}{i_t} \text{ (km/h)}$$
A continuously variable transmission (CVT) has a gear ratio that can be varied continuously within a certain range, thus providing an infinity of gear ratios. The transmission ratio is a function of the two effective diameters:

\[ i_s = \frac{D_2}{D_1} \]

where \( D_1 \) and \( D_2 \) are the effective diameters of the output pulley and input pulley, respectively.
Vehicle Performance

The performance of a vehicle is usually described by its maximum cruising speed, gradeability, and acceleration.
Maximum Speed of a Vehicle

The maximum speed of a vehicle is defined as the constant cruising speed that the vehicle can develop with full power plant load (full throttle of the engine or full power of the motor) on a flat road. The tractive effort and resistance equilibrium can be expressed as

\[ \frac{T_p}{r_d} = M g \frac{f_r}{r_d} \cos \alpha + \frac{1}{2} \rho C_D A \frac{V^2}{r_d} \]

The maximum speed of the vehicle can be written as

\[ V_{max} = \frac{\pi n_{p \max} r_d}{30 i_0 i_{g \min}} \text{(m/s)} \]

where \( n_{p \max} \) and \( i_{g \min} \) are the maximum speed of the engine (electric motor) and the minimum gear ratio of the transmission, respectively.
Gradeability

Gradeability is usually defined as the grade (or grade angle) that the vehicle can overcome at a certain constant speed, for instance, the grade at a speed of 100 km/h (60 mph). For heavy commercial vehicles or off-road vehicles, the gradeability is usually defined as the maximum grade or grade angle in the whole speed range.

When the vehicle drives on a road with relative small grade and constant speed, the tractive effort and resistance equilibrium can be written as

\[
\frac{T_p i_0 i_s \eta_l}{r_d} = M_v g f_r + \frac{1}{2} \rho_a C_D A_f V^2 + M_v g i
\]

Thus,

\[
i = \frac{(T_p i_0 i_s \eta_l/r_d) - M_v g f_r - (1/2) \rho_a C_D A_f V^2}{M_v g} = d - f_r
\]

where

\[
d = \frac{F_l - F_w}{M_v g} = \frac{(T_p i_0 i_s \eta_l/r_d) - (1/2) \rho_a C_D A_f V^2}{M_v g}
\]

is called the performance factor. While the vehicle drives on a road with a large grade, the gradeability of the vehicle can be calculated as

\[
\sin \alpha = \frac{d - f_r \sqrt{1-d^2+f_r^2}}{1+f_r^2}
\]
Acceleration Performance

The acceleration performance of a vehicle is usually described by its acceleration time and the distance covered from zero speed to a certain high speed (zero to 96 km/h or 60 mph, for example) on level ground. Using Newton’s second law, the acceleration of the vehicle can be written as

$$a = \frac{dV}{dt} = \frac{F_t - F_f - F_w}{M_v \delta} = \frac{(T_F i_0 i_s \eta_s/r_d) - M_v g f_r - (1/2) \rho_a C_D A_j V^2}{M_v \delta} = \frac{g}{\delta} (d - f_r)$$

where $\delta$ is called the mass factor. The mass factor can be written as

$$\delta = 1 + \frac{I_w}{M_v r_d^2} = \frac{i_0^2 i_s^2 I_p}{M_v r^2}$$

where $I_\omega$ is the total angular moment of the wheels and $I_p$ is the total angular moment of the rotating components associated with the power plant. Calculation of the mass factor, $\delta$, requires knowing the values of the mass moments of inertia of all the rotating parts. In the case where these values are not known, the mass factor, $\delta$, for a passenger car would be estimated using the following empirical relation:

$$\delta = 1 + \delta_1 + \delta_2 i_s^2 i_0^2$$
From equation $a=\frac{dV}{dt}$, the acceleration time, $t_a$, and distance, $S_a$, from low speed $V_1$ to high speed $V_2$ can be written, respectively, as

$$t_a = \int_{V_1}^{V_2} \frac{M_p \delta V}{(T_p i_g i_0 \eta_t / r_d) - M_v g f_r - (1/2) \rho_a C_D A_f V^2} \, dV$$

$$S_a = \int_{V_1}^{V_2} \frac{M_p \delta}{(T_p i_g i_0 \eta_t / r_d) - M_v g f_r - (1/2) \rho_a C_D A_f V^2} \, dV$$
Acceleration time and distance along with vehicle speed for a gasoline engine-powered passenger car with four-gear transmission.

Acceleration time and distance along with vehicle speed for an electric machine-powered passenger car with single-gear transmission.
Operating Fuel Economy

The fuel economy of a vehicle is evaluated by the amount of fuel consumption per 100 km traveling distance (liters/100 km) or mileage per gallon fuel consumption (miles/gallon), which is currently used in the U.S. The operating fuel economy of a vehicle depends on a number of factors, including fuel consumption characteristics of the engine, gear number and ratios, vehicle resistance, vehicle speed, and operating conditions.

Fuel Economy Characteristics of Internal Combustion Engines

The fuel economy characteristic of an internal combustion engine is usually evaluated by the amount of fuel per kWh energy output, which is referred to as the specific fuel consumption (g/kWh). The typical fuel economy characteristic of a gasoline engine is shown in Figure. For instance, when the engine shown in the Figure has a power output of 40 kW, its minimum specific fuel consumption would be 270 g/kWh at a speed of 2080 rpm.

Fuel economy characteristics of a typical gasoline engine
Calculation of Vehicle Fuel Economy

Vehicle fuel economy can be calculated by finding the load power and the specific fuel consumption of the engine. The engine power output is always equal to the resistance power of the vehicle, that is,

\[ P_e = \frac{V}{\eta_t} \left( F_f + F_w + F_g + M_v \delta \frac{dV}{dt} \right) \]

\[ P_e = \frac{V}{1000 \eta_t} \left( M_v g_f r \cos \alpha + \frac{1}{2} \rho_a C_D A_f V^2 + M_v g \sin \alpha + M_v \delta \frac{dV}{dt} \right) \text{(kW)} \]

The engine speed, related to vehicle speed and gear ratio, can be expressed as

\[ N_e = \frac{30 V i_g i_0}{\pi r_d} \]

The time rate of fuel consumption can be calculated by

\[ Q_{fr} = \frac{P_e g_e}{1000 \gamma_f} \text{ (l/h)} \]

where \( g_e \) is the specific fuel consumption of the engine in g/kWh and \( \gamma_f \) is the mass density of the fuel in kg/l. The total fuel consumption within a total distance, \( S \), at a constant cruising speed, \( V \), is obtained by

\[ Q_s = \frac{P_e g_e S}{1000 \gamma_f V} \]
The Figure shows an example of the fuel economy characteristics of a gasoline vehicle at constant cruising speed on level ground. This figure indicates that at high speeds, the fuel consumption increases because the aerodynamic resistance power increases with the speed cubed. This figure also indicates that with a high-speed gear (small gear ratio), the fuel economy of the vehicle can be enhanced due to the reduced engine speed at a given vehicle speed and increased gear ratio.
The Figure shows the operating points of an engine at constant vehicle speed, with the highest gear and the second highest gear. It indicates that the engine has a much lower operating efficiency in low gear than in high gear. This is the reason why the fuel economy of a vehicle can be improved with more gear transmission and continuous variable transmission.
It should be noted that because of the complexity of vehicle operation in the real world, fuel consumption at constant speed cannot accurately represent fuel consumption for a vehicle under real driving conditions. Thus, various drive cycles have been developed to simulate. The drive cycles are usually represented by the speed of the vehicle along with the relative driving time. The Figure shows the urban and highway drive cycles of EAP FTP75 used in the U.S.
To calculate fuel consumption in a drive cycle, the total fuel consumption can be obtained by the summation of fuel consumption in each time interval, $\Delta t_i$,

$$Q_{tc} = \sum_{i} \frac{P_{ei} g_{ei}}{1000} \Delta t_i$$

where $P_{ei}$ is the average power of the engine in the $i_{th}$ time interval in kW, $g_{ei}$ is the average specific fuel consumption of the engine in the $i_{th}$ time interval in g/kWh, and $\Delta t_i$ is the $i_{th}$ time interval in h.
Basic Techniques to Improve Vehicle Fuel Economy

The effort to improve the fuel economy of vehicles has been an ongoing process in the automobile industry. Fundamentally, the techniques used include the following aspects:

(1) Reducing vehicle resistance
(2) Improving engine operation efficiency
(3) Properly matched transmission
(4) Advanced drive trains
Braking Performance

The braking performance of a vehicle is undoubtedly one of the most important characteristics that affect vehicle safety. In urban driving, a significant amount of energy is consumed in braking.

Braking Force

The Figure shows a wheel during braking. The brake pad is pressed against the brake plate, thus developing a frictional torque on the brake plate.

(a) Braking torque and braking force, and (b) relationship between braking torque and braking force.
The braking force can be expressed as

\[ F_b = \frac{T_b}{r_d} \]

A maximum braking force limited by the adhesive capability can be expressed as

\[ F_{b_{\text{max}}} = \mu_b W \]

where \( \mu_b \) is the adhesive coefficient of the tire–ground contact.
Braking Distribution on Front and Rear Axles

The Figure shows the forces acting on a vehicle during braking on a flat road. Rolling resistance and aerodynamic drag are ignored in this figure, because they are quite small compared to the braking forces. $j$ is the deceleration of the vehicle during braking, which can be easily expressed as

$$j = \frac{F_{bf} + F_{br}}{M_v}$$

where $F_{bf}$ and $F_{br}$ are the braking forces acting on front and rear wheels, respectively.
By considering the equilibrium of moments about the front and rear tire–ground contact points A and B, the normal loads on the front and rear axles, $W_f$ and $W_r$, can be expressed as

$$W_f = \frac{M_v g}{L} \left( L_b + h_g \frac{j}{g} \right)$$

$$W_r = \frac{M_v g}{L} \left( L_a - h_g \frac{j}{g} \right)$$

where $j$ is the deceleration of the vehicle.

The braking forces of the front and rear axle should be proportional to their normal load, respectively; thus, one obtains

$$\frac{F_{bf}}{F_{br}} = \frac{W_f}{W_r} = \frac{L_b + h_g j}{L_a - h_g j} \frac{g}{g}$$

In vehicle design, the actual braking forces on the front and rear axle are usually designed to have a fixed linear proportion. This proportion is represented by the ratio of the front axle braking force to the total braking force of the vehicle, that is

$$\beta = \frac{F_{bf}}{F_b}$$
where $F_b$ is the total braking force of the vehicle ($F_b = F_{bf} + F_{br}$). With $\beta$ being the actual braking force on the front and rear axle, this can be expressed as

$$F_{bf} = \beta F_b$$

$$F_{br} = (1 - \beta)F_b$$

Thus, one obtains

$$\frac{F_{bf}}{F_{br}} = \frac{\beta}{1 - \beta}$$
Figure shows the ideal and actual braking force distribution curves (labeled $I$ and $\beta$ curves). It is clear that only one intersection point exists, at which the front and rear axles lock up at the same time. This point represents one specific road adhesive coefficient, $\mu_0$. 

\[ \frac{\beta}{1-\beta} = \frac{L_b + \mu_0 h_s}{L_a - \mu_0 h_s} \]

\[ \mu_0 = \frac{L\beta - L_b}{h_s} \]

\[ \beta = \frac{\mu_0 h_s + L_b}{L} \]
When the rear wheels lock up first, the vehicle will lose directional stability, as shown in loss of directional stability Figure. The figure shows the top view of a two-axle vehicle acted upon by the braking force and the inertia force.

Loss of directional stability due to the lockup of rear wheels

Angular deviation of a car when all four wheels do not lock at the same instant